UNIVERSITY OF JOHANNESBURG

FACULTY OF SCIENCE

DEPARTMENT OF MATHEMATICS
BACCALAUREUS TECHNOLOGIA
ENGINEERING: Electrical

MODULE MAT1AW4
ENGINEERING MATHEMATICS 4

CAMPUS **DFC**

JUNE EXAMINATION 2016

DATE: 31 MAY 2016 **SESSION: 12:30 – 15:30**

ASSESSOR: **MRS H. Kotze**

MODERATOR: **Mr. V. Makhoshi**

DURATION: 3 hours **MARKS: 100**

NUMBER OF PAGES: **3 PAGES**

INSTRUCTIONS: **Calculators are allowed**
All answers must be completed in exam script

REQUIREMENTS: **Mathematics information booklet**
Answer scripts

SECTION A: Z-TRANSFORMATIONS

QUESTION 1 Determine the following z-transforms:

1.1 If $x(k) = \begin{cases} 0 & k = 0; k = \text{odd} \\ 2 & k = 2, 6, 10, \ldots \\ -2 & k = 4, 8, 12, \ldots \end{cases}$;

1.1.1 Find $z\{x(k)\}$

1.1.2 Find $z\{x(k+2)\}$. (7)

1.2 Find $z\left\{\dfrac{2-p}{p^2(p+1)}\right\}$ by using partial fractions. (6)

1.3 Find $z\{k\,e^{2-k}\}$. (3)

1.4 If $x(k) = k - 1$; find $z\{x(k-1)\}$. (3)

[19]

QUESTION 2 Determine the inverse z-transforms using methods as indicated:

2.1 If $x(k) = z^{-1}\left\{\dfrac{z^3}{4z^4 - z^3 - 3z^2}\right\}$

2.1.1 Use partial fractions to find $x(k)$.

2.1.2 Now calculate $x(1)$. (7)

2.2 If $x(k) = z^{-1}\left\{\dfrac{z}{(z+1)^4}\right\}$, use the residue method find $x(k)$. (5)

2.3 If $x(k) = z^{-1}\left\{\dfrac{z^2}{z^2 - z - 1}\right\}$, use the power series method to find $x(3)$ and $x(4)$. (6)

[18]

QUESTION 3 Use z-transform methods to answer the following:

3.1 Given the difference equation $x(k) - x(k-1) - 12x(k-2) = y(k)$ where $y(k) = \begin{cases} 0 & k \neq 0; k \neq 1 \\ 1 & k = 0; k = 1 \end{cases}$. Find an expression for $x(k)$. (6)

3.2 Given the forward difference equation $6x(k+2) - x(k+1) - 2x(k) = 0$ with initial data $x(0) = 0$ and $x(1) = 7$

3.2.1 Find an expression for $x(k)$.

3.2.2 Now calculate $x(2)$ and $x(3)$. (8)

[14]

TOTAL SECTION A: 51

SECTION B PARTIAL DIFFERENTIAL EQUATIONS

QUESTION 4

Solve $\dfrac{\partial^2 u(x,y)}{\partial y^2} = \sqrt{x\,y}$ if $u(x,0)=1$ and $u(x,1)=0.$ **[10]**

QUESTION 5

Given the one-dimensional heat equation $\dfrac{\partial^2 u(x,t)}{\partial x^2} = \dfrac{1}{k}\dfrac{\partial u(x,t)}{\partial t}$ subject to the initial temperature $u(x,0) = f(x) = \begin{cases} x & 0 \le x < 3 \\ 4 - \dfrac{2}{3}x & 3 \le x \le 6 \end{cases}$. The two ends of the bar are kept at zero temperature for all values of *t* so that the boundary conditions are $u(0,t) = u(6,t) = 0$ for $t \ge 0$. By ***separation of the variables***:

5.1 Show that the general solution is given by

$u(x,t) = (A\cos\lambda x + B\sin\lambda x)Ce^{-\lambda^2 k t}$.

5.2 Determine the unique solution. **[25]**

QUESTION 6

Solve the one dimensional heat equation $2\dfrac{\partial^2 u(x,t)}{\partial x^2} = \dfrac{\partial u(x,t)}{\partial t}$ if $x > 0, t > 0$ by using ***Laplace transforms*** with respect to *t* subject to the conditions $u(x,0) = 2\sin 4\pi x$ for a bar of length $0 < x < 4$ where $u(0,t) = u(4,t) = 0$. **[15]**

TOTAL SECTION B: 50